PART ONE : 2 Questions for 30 Points

1) (16 points) From the group of top- eight NDU graduates for the year 2012 of which 3 are females, 4 students were randomly selected. Let X be a random variable representing the number of males among the selected 4 students.

a) Find the probability distribution function f(x) of X (8 points)

3 females 5 Males

$X = 1, 2, 3, 4$

X	1	2	3	4
	$\frac{5}{70}$	$\frac{30}{70}$	$\frac{30}{70}$	$\frac{5}{70}$

$C_4^8 = 70$ ation

$P(X=0) = \frac{C_0^5 \cdot C_4^3}{70} = \frac{1}{70}$

$P(X=1) = \frac{C_1^5 \cdot C_3^3}{70} = \frac{20}{70}$

$P(X=2) = \frac{C_2^5 \quad C_2^3}{70} =$

$C_x^5 \, C_{4-x}^3$

$f(x) =$

b) Find the expected value of X (3 points)

$E(x) = x f(x) =$ $+ \left(1 \times \frac{5}{70}\right) + \left(2 \times \frac{30}{70}\right) + \left(3 \times \frac{30}{70}\right) + \left(4 \times \frac{5}{70}\right)$

$=$

c) The NDU president Dr. W. Moussa decides to pay \$500 to each male, and \$1000 to each female among the selected 4. What is the expected amount that Dr. Moussa will pay? (5 points)

3

2) (14 points) The profit " in $1000 " of a contractor is a continuous random variable X, With probability density function

$$f(x) = \begin{cases} \frac{1}{18}(x+1) \text{ for } -1 \le x \le 5 \\ 0 \quad \text{otherwise} \end{cases}$$

a) Find the expected profit (4 points)

$\int_{-1}^{5} x f(x) = \int_{-1}^{5} x \times \frac{1}{18}(x+1)\,dx = \frac{1}{18}\int_{-1}^{5}(x^2+x)\,dx = \frac{1}{18}\left[\frac{x^3}{3}+\frac{x^2}{2}\right]_{-1}^{5}$ $= \frac{54}{18} = 3$

$= \int_{-\infty}^{\infty} x f(x) = \int_{-\infty}^{-1} 0\,dx + \int_{-1}^{5} x f(x)\,dx + \int_{5}^{\infty} 0\,dx$

$\boxed{E(x) = 54}$

$\Rightarrow E(\text{profit}) = 3000$

b) What is the probability of having a negative profit? (3 points)

?

c) Find the cumulative distribution function F(x), then use it to find the probability of making a profit of $3000 or more. (7 points)

$-1 \quad = \emptyset(x) = 0$

$/5$

$\int_{-1}^{b} f(x)\,dx = \frac{1}{18}\left[\frac{x^2}{2}+x\right]_{-1}^{b}$

$= \frac{1}{18}\left[\frac{b^2}{2}+b-\frac{1}{2}+1\right]$

$= 1 \, (\ldots$

what's

40

PART TWO : 15 Questions for 75 Points

For each of the following questions, circle the right answer. There is only <u>one</u> correct answer for each question. If you circle more than one answer per question, your answer would be considered incorrect

*** Let X be a geometric random variable. If $P(X>3) = 0.25$, then $P(X>6 \mid X>3)$ equals

a) 0.25 b) 0.5 c) 0.0625 d) None of these

$P(X>6/X>3) = \frac{P(X>6, X>3)}{P(X>3)} = \frac{\quad}{0.25} =$

X A

*** In a certain mall, the probability that we spend more than 4 minutes to serve a customer is 0.4. If five customers are waiting for the service, then the probability that the service time of exactly 2 of them is more than 4 minutes is

a) 0.0576 b) 0.576 c) 0.16 d) None of these

X D

*** Given X and Y two random variables, with $Y = 22 - 3X$. If $\mu_X = 7$ and $\sigma_X = 2$, then

a) $\mu_Y = 1$ and $\sigma_Y = -6$

b) $\mu_Y = 43$ and $\sigma_Y = 6$

c) $\mu_Y = 1$ and $\sigma_Y = 6$

d) $\mu_Y = 1$ and $\sigma_Y = 36$

X C

$E(Y) = E(22 - 3X) = -3E(X) + 22 = 1$

$V(Y) = V(22 - 3X) = -3V(X) = -6$

*** A and B are two events, If P (A.B) = 0.1 and P(AUB)= 0.8, then $P(\bar{A}.\bar{B})$ is equal to

a) 0.9 b) 0.2 c) 0.1 d) None of these

$P(\bar{A}.\bar{B}) = 1 - P(\bar{A}\cup\bar{B}) = 1\ 0.8 = 0.2$

$P(A\cup B) = P(A) + P($

*** If the cumulative distribution function F(x) of a continuous random variable X is given by

$$F(x) = \begin{cases} \frac{1}{56}(x^2 - x) & \textit{for } 1 \le x \le 8 \\ 1 & \textit{for } x > 8 \\ 0 & \textit{else where} \end{cases}$$

XB

then P(X> 4) equals to

a) $\frac{42.5}{56}$ b) $\frac{44}{56}$ c) $\frac{13.5}{56}$ d) None of these

$P(X>4) = 1 - P(X\le 4)$

$P(X\le 4) = \int_{-\infty}^{4} f(x)\,dx = \int_{-\infty}^{1} 0\,dx + \int_{1}^{4} \frac{1}{56}(x^2 - x)\,dx + 0$

$= \frac{1}{56}\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^4 = \frac{13.5}{56}$

*** If X is a Poisson random variable with a mean of 4, then P(X=2) equals to

a) $4e^{-4}$ b) $2e^{-2}$ c) $8e^{-4}$ d) None of these

$\frac{e^{-4} \times 4^2}{2!} = 8e^{-4}$

*** A system consists of two components A and B that operate independently. The probability that component A fails is 0.01, and that component B fails is 0.02. What is the probability that exactly one of these two components will fail?

a) 0.0296 b) 0.0098 c) 0.0198 d) None of these

P(fail A) = 0.01 P(fail B) = 0.02

0.01 0.

0.01 0.02

0.01 X 0.98 +

*** A company uses two different machines to produce same certain item. The company knows that usually 10% of the items produced by the first machine are defective , and of those produced by the second machine 8% are defective. On certain day 75% of the production was produced by the first machine. If one item was randomly selected, then the probability it is defective is

a) 0.085 b) 0.09 c) 0.095 d) None of these

2 machines

10% def first mach

8% 2nd

25% 2nd mach

75% 1st m

$0.25 \times 0.08 + 0.75 \times 0.1 = 0.095$

*** A large lot of items of which 60% were produced by machine I, and 40% by machine II, contains 7% defectives. It is known that 5% of the items produced by machine I are defectives and of those produced by machine II 10% are defectives. One item was randomly selected, find the probability that it was produced by machine II, given that it was defective.

a) 0.429 b) 0.571 c) 0.1 d) None of these

X B

60% mach I

40% II

7% def.

5% def

10% def

***Let X " the number of customers arrive at a certain small bank" follows a poisson distribution with λ= 20/hour. At 9:00 Am, one customer arrives, what is the probability that the next customer will arrive after 9:05 Am?

a) e^{-3} b) $e^{-1.667}$ c) e^{-5} d) None of these

5 min 0 custs.

$\lambda = 20/\text{hour} = \frac{1}{3}/\text{min}$

$\frac{e^{-\frac{1}{3}\times 5} \lambda^0}{0!} = e^{-1.662}$

X A

*** A and B are two independent events. If P(A.B) = 0.08 and P (B|A) = 0.8, then P (A U B) equals to

a) 0.82 b) 0.96 c) 1 d) None of these

$P(A \cup B) = P(A) + P(B) - P(A.B)$

$P(B/A) = \frac{P(A.B)}{P(A)}$

$P(A/B)$

$P_{(A)} = \frac{0.08}{0.8} = 0.1$

*** Suppose you are throwing a fair die. The probability of getting number 6 for the first time on the 3-rd trial is

a) $\frac{10}{216}$ b) $\frac{25}{216}$ c) $\frac{50}{216}$ d) None of these

$C_{m-1}^{x-1} \, p^{r} q^{x-r}$

$p = \frac{1}{6}$ $q = \frac{5}{6}$ $x = 3$ $m = 1$

$C_0^2 \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^2 = 25/216$

*** Suppose you are throwing a fair die two times. What is the probability that the second trial gives number 3 given that the first gave number 3?

a) $\frac{1}{2}$ b) $\frac{1}{36}$ c) $\frac{35}{36}$ d) None of these

XD

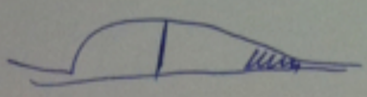
$\frac{1}{6} \times \frac{1}{6}$

*** The amount X in c.c. of beverage that a machine automatically dispenses into 260 c.c. cups is a normal random variable with mean 253. 4 c.c. If 4.95 % of the cups overflow, then the standard deviation of X is

a) 5 b) 6 c) 4 d) None of these

$\frac{260 - 253.4}{\sigma} = \frac{6.6}{\sigma}$

$\frac{6.6}{\sigma} = 1.65$ $\sigma = 4$



$0.5 - 0.0495 = 0.4505$ $P(0.4505) = 1.65$

*** Let X be a non-negative random variable (i.e. X≥ 0). If $\mu_X = 6$, and $\sigma_X = 4.$, then $P(X \leq 14)$ is

a) At least 0.75 b) At most 0.25 c) at least 0.55 d) None of these

$6 - 4K = 14$ $K = -2$

$1 - \frac{1}{K^2} = 0{,}75$